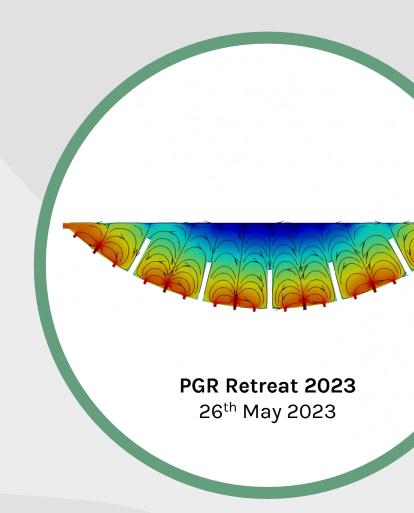
Placental haemodynamics: Transport effects at the organ scale

Adam Blakey (he/him)

Penny Gowland, Paul Houston, Matthew Hubbard, George Hutchinson, Lopa Leach, and Reuben O'Dea



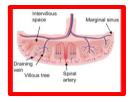




Today's talk



Introduction

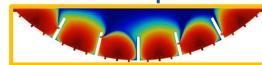




Blood flow



Nutrient transport

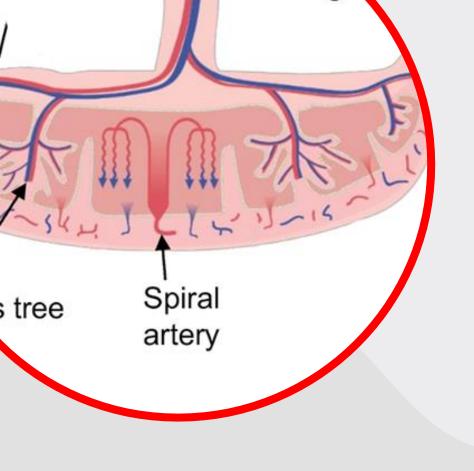




In progress

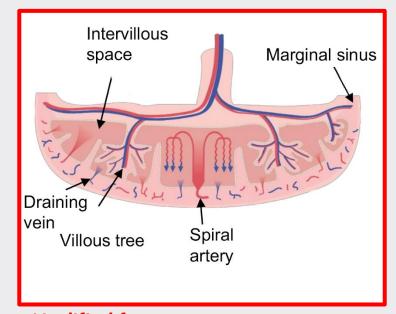












Modified from [Dellschaft et al., 2020]



- Provides nutrients and oxygen to foetus
- Removes waste products
- Vital to fetal development

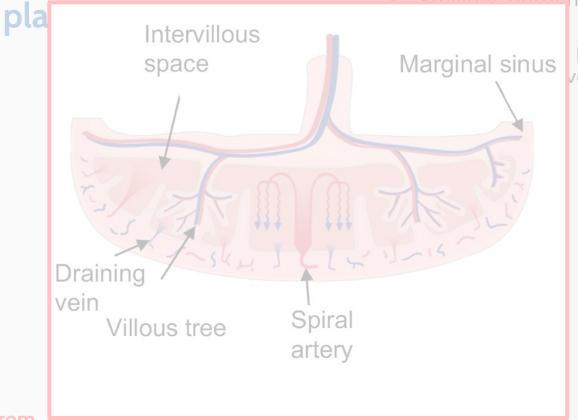


What is a



Provides putrients and oxygen to

products velopment



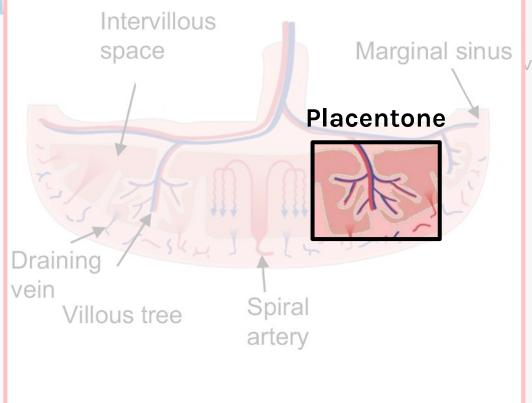
Modified from



What is a



Provides putrients and oxygen to



products velopment

Modified from [Dellschaft et al., 2020]













[Lopa Leach]



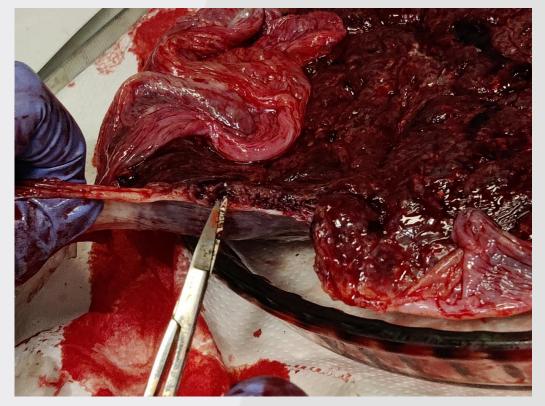




[Lopa Leach]



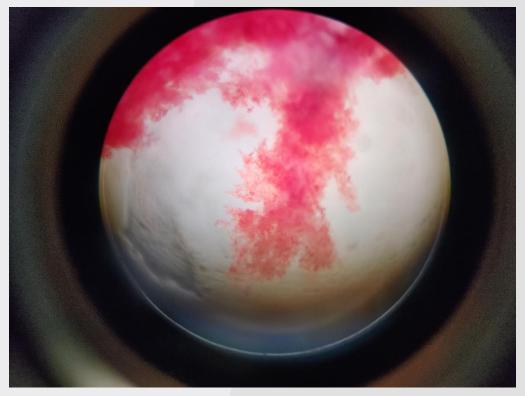




[Lopa Leach]



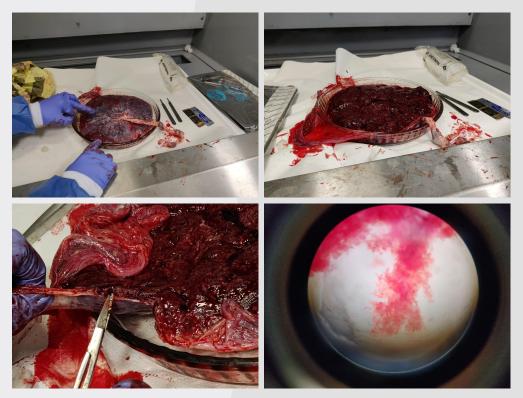




[Lopa Leach]



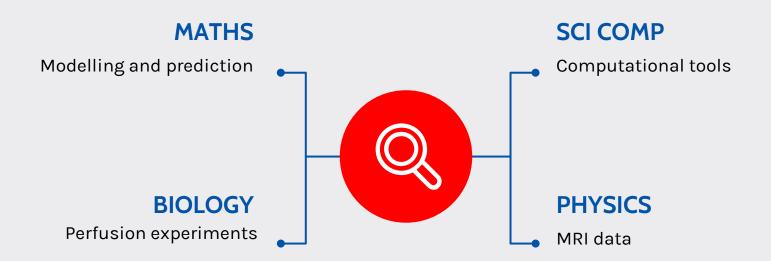




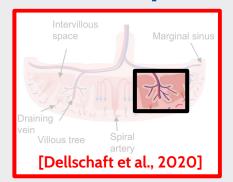
[Lopa Leach]

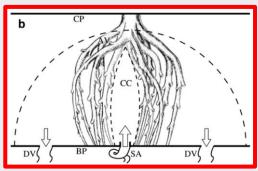


Inter-disciplinary Work









[Chernyavsky et al., 2010]



- Homogenise tree structure
 - → porous medium
- Incompressible flow
- Navier-Stokes & Brinkman

Brinkman:

$$-\nabla^{2} \boldsymbol{u} + \frac{1}{\operatorname{Dr}} \boldsymbol{u} + \nabla p = f_{\mathrm{B}}$$

$$\nabla \cdot \boldsymbol{u} = 0$$

$$\boldsymbol{u}_{\mathrm{wall}} = \mathbf{0}$$

$$\boldsymbol{u}_{\mathrm{neumann}} \equiv (\nabla \boldsymbol{u} - p\underline{\boldsymbol{I}}) \cdot \boldsymbol{n} = \mathbf{0}$$

Navier-Stokes:

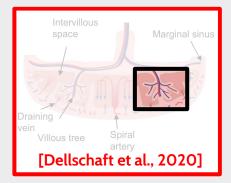
$$-\nabla^{2} \boldsymbol{u} + \operatorname{Re} \nabla \cdot (\boldsymbol{u} \otimes \boldsymbol{u}) + \nabla p = f_{\text{NS}}$$

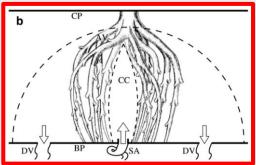
$$\nabla \cdot \boldsymbol{u} = 0$$

$$\boldsymbol{u}_{\text{inlet}} = \frac{(R^{2} - r^{2})}{R^{2}} \hat{\boldsymbol{y}}$$

$$\boldsymbol{u}_{\text{Wall}} = \boldsymbol{0}$$







[Chernyavsky et al., 2010]



- Homogenise tree structure
 - → porous medium
- Incompressible flow
- Navier-Stokes & Brinkman

Navier-Stokes-Brinkman:

$$-\nabla^{2} \mathbf{u} + \operatorname{Re} \nabla \cdot (\mathbf{u} \otimes \mathbf{u}) + \frac{1}{\operatorname{Dr}} \mathbf{u} + \nabla p = f$$

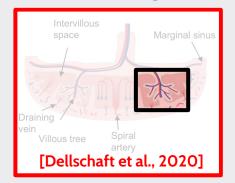
$$\nabla \cdot \mathbf{u} = 0$$

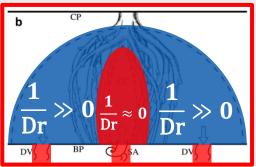
$$\mathbf{u}_{\text{inlet}} = \frac{(R^{2} - r^{2})}{R^{2}} \hat{\mathbf{y}}$$

$$\mathbf{u}_{\text{wall}} = \mathbf{0}$$

$$\mathbf{u}_{\text{neumann}} \equiv (\nabla \mathbf{u} - p \mathbf{I}) \cdot \mathbf{n} = \mathbf{0}$$







[Chernyavsky et al., 2010]



- Homogenise tree structure
 - → porous medium
- Incompressible flow
- Navier-Stokes & Brinkman

Navier-Stokes-Brinkman:

$$-\nabla^{2} \boldsymbol{u} + \operatorname{Re} \nabla \cdot (\boldsymbol{u} \otimes \boldsymbol{u}) + \frac{1}{\operatorname{Dr}(\boldsymbol{x})} \boldsymbol{u} + \nabla p = f$$

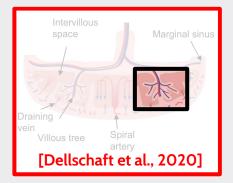
$$\nabla \cdot \boldsymbol{u} = 0$$

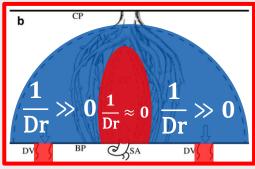
$$\boldsymbol{u}_{\text{inlet}} = \frac{(R^{2} - r^{2})}{R^{2}} \widehat{\boldsymbol{y}}$$

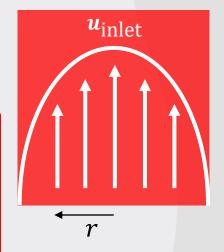
$$\boldsymbol{u}_{\text{wall}} = \boldsymbol{0}$$

$$\boldsymbol{u}_{\text{neumann}} \equiv (\nabla \boldsymbol{u} - p\underline{\boldsymbol{I}}) \cdot \boldsymbol{n} = \boldsymbol{0}$$











- → porous medium
- Incompressible flow
- Navier-Stokes & Brinkman

Navier-Stokes-Brinkman:

$$-\nabla^{2} \mathbf{u} + \operatorname{Re} \nabla \cdot (\mathbf{u} \otimes \mathbf{u}) + \frac{1}{\operatorname{Dr}(\mathbf{x})} \mathbf{u} + \nabla p = f$$

$$\nabla \cdot \mathbf{u} = 0$$

$$\mathbf{u}_{\text{inlet}} = \frac{(R^{2} - r^{2})}{R^{2}} \hat{\mathbf{y}}$$

$$\mathbf{u}_{\text{wall}} = \mathbf{0}$$

$$\mathbf{u}_{\text{neumann}} \equiv (\nabla \mathbf{u} - p\underline{\mathbf{I}}) \cdot \mathbf{n} = \mathbf{0}$$

[Chernyavsky et al., 2010]



DGFEMdiscretisation

Find
$$\forall \boldsymbol{u}_h, p_h \in \boldsymbol{V}_h \times Q_h$$
 s.t.
$$A(\boldsymbol{u}_h, \boldsymbol{v}_h) + B(\boldsymbol{v}_h, p_h) - B(\boldsymbol{u}_h, q_h) + C(\boldsymbol{u}_h, \boldsymbol{v}_h) = F(\boldsymbol{v}_h) - G(q_h),$$
 $\forall \boldsymbol{v}_h, q_h \in \boldsymbol{V}_h \times Q_h.$

[Cliffe et al., 2010]

Navier-Stokes-Brinkman:

$$-\nabla^{2} \mathbf{u} + \operatorname{Re} \nabla \cdot (\mathbf{u} \otimes \mathbf{u}) + \frac{1}{\operatorname{Dr}(\mathbf{x})} \mathbf{u} + \nabla p = f$$

$$\nabla \cdot \mathbf{u} = 0$$

$$\mathbf{u}_{\text{inlet}} = \frac{(R^{2} - r^{2})}{R^{2}} \hat{\mathbf{y}}$$

$$\mathbf{u}_{\text{wall}} = \mathbf{0}$$

$$\mathbf{u}_{\text{neumann}} \equiv (\nabla \mathbf{u} - p\underline{\mathbf{I}}) \cdot \mathbf{n} = \mathbf{0}$$

$$A(\boldsymbol{u}, \boldsymbol{v}) = \int_{\Omega} \nabla_{h} \boldsymbol{u} : \nabla_{h} \boldsymbol{v} - \int_{F \cup \Gamma_{D}} \{ \{ \nabla_{h} \boldsymbol{v} \} \} : [\![\boldsymbol{u}]\!] + \{ \{ \nabla_{h} \boldsymbol{u} \} \} : [\![\boldsymbol{v}]\!]$$

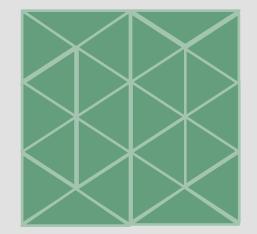
$$+ \int_{F \cup \Gamma_{D}} \sigma[\![\boldsymbol{u}]\!] : [\![\boldsymbol{v}]\!] + \frac{1}{\mathrm{Dr}} \int_{\Omega} \boldsymbol{u} \cdot \boldsymbol{v}$$

$$B(\boldsymbol{u}, q) = -\int_{\Omega} q \nabla_{h} \cdot \boldsymbol{v} + \int_{F \cup \Gamma_{D}} \{ \{q\} \} [\![\boldsymbol{v}]\!]$$

$$C(\boldsymbol{u}, \boldsymbol{v}) = -\int_{\Omega} (\boldsymbol{u} \otimes \boldsymbol{u}) : \nabla_{h} \boldsymbol{v} + \int_{F \cup \Gamma_{D}} \mathcal{H}(\boldsymbol{u}^{+}, \boldsymbol{u}^{-}, \boldsymbol{n}) [\![\boldsymbol{v}]\!]$$

$$F(\boldsymbol{v}) = \int_{\Gamma_{D}} \sigma \boldsymbol{g}_{D} \cdot \boldsymbol{v} - \int_{\Gamma_{D}} (\boldsymbol{g}_{D} \otimes \boldsymbol{n}) : (\nabla_{h} \boldsymbol{v})$$

$$G(q) = \int_{\Gamma} q \boldsymbol{g}_{D} \cdot \boldsymbol{n}$$





DGFEM discretisation

Find
$$\forall \boldsymbol{u}_h, p_h \in \boldsymbol{V}_h \times Q_h$$
 s.t.
 $A(\boldsymbol{u}_h, \boldsymbol{v}_h) + B(\boldsymbol{v}_h, p_h) - B(\boldsymbol{u}_h, q_h)$
 $+ C(\boldsymbol{u}_h, \boldsymbol{v}_h) = F(\boldsymbol{v}_h) - G(q_h),$
 $\forall \boldsymbol{v}_h, q_h \in \boldsymbol{V}_h \times Q_h.$

[Cliffe et al., 2010]

Navier-Stokes-Brinkman:

$$-\nabla^{2} u + \operatorname{Re} \nabla \cdot (u \otimes u) + \frac{1}{\operatorname{Dr}(x)} u + \nabla p = f$$

$$\nabla \cdot u = 0$$

$$u_{\text{inlet}} = \frac{(R^{2} - r^{2})}{R^{2}} \widehat{y}$$

$$u_{\text{wall}} = 0$$

$$u_{\text{neumann}} \equiv (\nabla u - p\underline{I}) \cdot n = 0$$

$$A(\boldsymbol{u}, \boldsymbol{v}) = \int_{\Omega} \nabla_{h} \boldsymbol{u} : \nabla_{h} \boldsymbol{v} - \int_{F \cup \Gamma_{D}} \{ \{ \nabla_{h} \boldsymbol{v} \} \} : [\![\boldsymbol{u}]\!] + \{ \{ \nabla_{h} \boldsymbol{u} \} \} : [\![\boldsymbol{v}]\!]$$

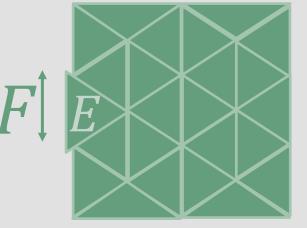
$$+ \int_{F \cup \Gamma_{D}} \sigma[\![\boldsymbol{u}]\!] : [\![\boldsymbol{v}]\!] + \frac{1}{\mathrm{Dr}} \int_{\Omega} \boldsymbol{u} \cdot \boldsymbol{v}$$

$$B(\boldsymbol{u}, q) = -\int_{\Omega} q \nabla_{h} \cdot \boldsymbol{v} + \int_{F \cup \Gamma_{D}} \{ \{q\} \} [\![\boldsymbol{v}]\!]$$

$$C(\boldsymbol{u}, \boldsymbol{v}) = -\int_{\Omega} (\boldsymbol{u} \otimes \boldsymbol{u}) : \nabla_{h} \boldsymbol{v} + \int_{F \cup \Gamma_{D}} \mathcal{H}(\boldsymbol{u}^{+}, \boldsymbol{u}^{-}, \boldsymbol{n}) [\![\boldsymbol{v}]\!]$$

$$F(\boldsymbol{v}) = \int_{\Gamma_{D}} \sigma \boldsymbol{g}_{D} \cdot \boldsymbol{v} - \int_{\Gamma_{D}} (\boldsymbol{g}_{D} \otimes \boldsymbol{n}) : (\nabla_{h} \boldsymbol{v})$$

$$G(q) = \int_{\Gamma} q \boldsymbol{g}_{D} \cdot \boldsymbol{n}$$





DGFEM

 $A(\boldsymbol{u}, \boldsymbol{v}) = \int_{\Omega} \nabla_h \boldsymbol{u} : \nabla_h \boldsymbol{v} - \int_{F \cup \Gamma_D} \{ \{ \nabla_h \boldsymbol{v} \} \} : [\![\boldsymbol{u}]\!] + \{ \{ \nabla_h \boldsymbol{u} \} \} : [\![\boldsymbol{v}]\!] + \int_{F \cup \Gamma_D} \sigma[\![\boldsymbol{u}]\!] : [\![\boldsymbol{v}]\!] + \frac{1}{\mathrm{Dr}} \int_{\Omega} \boldsymbol{u} \cdot \boldsymbol{v}$

discretisation

Find $\forall \mathbf{u}_h, p_h \in \mathbf{V}_h \times Q_h$ $A(\mathbf{u}_h, \mathbf{v}_h) + B(\mathbf{v}_h, p_h)$ $+ C(\mathbf{u}_h, \mathbf{v}_h) = F(\mathbf{v}_h)$ $\forall \mathbf{v}_h, q_h \in \mathbf{V}_h \times Q_h$.

[Cliffe et al., 2010]

Navier-Stoke

$$-
abla^2 oldsymbol{u} + \operatorname{Re} oldsymbol{
abla} \cdot (oldsymbol{u} igotimes oldsymbol{u})$$

$$\nabla \cdot \boldsymbol{u} = 0$$

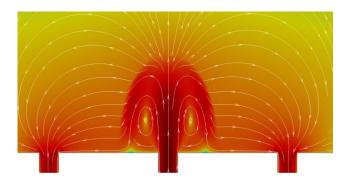
$$u_{\text{inlet}} = \frac{(R^2 - r^2)}{R^2} \hat{\mathbf{y}}$$

$$u_{
m wall}=0$$

$$u_{\text{neumann}} \equiv (\nabla u - p\underline{I}) \cdot n = 0$$

DGFEM motivation

Stable for large variation in parameters



[[ע

$$\mathcal{H}(u^+,u^-,n)[v]$$

$$\nabla_h \boldsymbol{v}$$





DGFEM

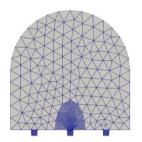
discretisation

$A(\boldsymbol{u},\boldsymbol{v}) = \int_{\Omega} \nabla_h \boldsymbol{u} : \nabla_h \boldsymbol{v} - \int_{F \cup \Gamma_D} \{\{\nabla_h \boldsymbol{v}\}\} : [\![\boldsymbol{u}]\!] + \{\{\nabla_h \boldsymbol{u}\}\} : [\![\boldsymbol{v}]\!]$

$+ \int_{F \cup \Gamma_D} \sigma \llbracket \boldsymbol{u} \rrbracket : \llbracket \boldsymbol{v} \rrbracket + \frac{1}{\operatorname{Dr}} \int_{\Omega} \boldsymbol{u} \cdot \boldsymbol{v}$

DGFEM motivation

- Stable for large variation in parameters
- Moving meshes and hyperbolic term



[Cliffe et al., 2010]

 $\forall \boldsymbol{v}_h, q_h \in \boldsymbol{V}_h \times Q_h$.

Navier-Stoke

$$-
abla^2 oldsymbol{u} + \operatorname{Re} oldsymbol{
abla} \cdot (oldsymbol{u} oldsymbol{\otimes} oldsymbol{u})$$

Find $\forall \mathbf{u}_h, p_h \in \mathbf{V}_h \times Q_h$ $A(\mathbf{u}_h, \mathbf{v}_h) + B(\mathbf{v}_h, p_h)$

 $+C(\boldsymbol{u}_h,\boldsymbol{v}_h)=F(\boldsymbol{v}_h)$

$$\nabla \cdot \boldsymbol{u} = 0$$

$$u_{\text{inlet}} = \frac{(R^2 - T^2)}{R^2} \hat{y}$$

$$u_{\mathrm{wall}} = 0$$

$$u_{\text{neumann}} \equiv (\nabla u - p\underline{I}) \cdot n = 0$$



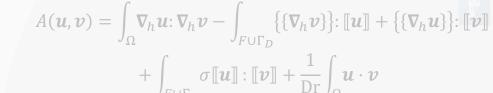
 $\mathcal{H}(\mathbf{u}^+,\mathbf{u}^-,\mathbf{n})[\![\mathbf{v}]\!]$





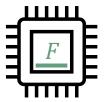
DGFEM

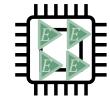
discretisation



DGFEM motivation

- Stable for large variation in parameters
- Moving meshes and hyperbolic term
- More parallelisable





Find $\forall \mathbf{u}_h, p_h \in \mathbf{V}_h \times Q_h$ $A(\mathbf{u}_h, \mathbf{v}_h) + B(\mathbf{v}_h, p_h)$

$$+ C(\boldsymbol{u}_h, \boldsymbol{v}_h) = F(\boldsymbol{v}_h)$$

$$\forall \boldsymbol{v}_h, q_h \in \boldsymbol{V}_h \times Q_h.$$

[Cliffe et al., 2010]

Navier-Stoke

$$-
abla^2 oldsymbol{u} + \operatorname{Re} oldsymbol{
abla} \cdot (oldsymbol{u} igotimes oldsymbol{u})$$

$$\nabla \cdot \boldsymbol{u} = 0$$

$$u_{\text{inlet}} = \frac{(R^2 - r^2)}{R^2} \widehat{y}$$

$$u_{\text{wall}} = 0$$

$$u_{\text{neumann}} \equiv (\nabla u - p\underline{I}) \cdot n = 0$$

v
bracket

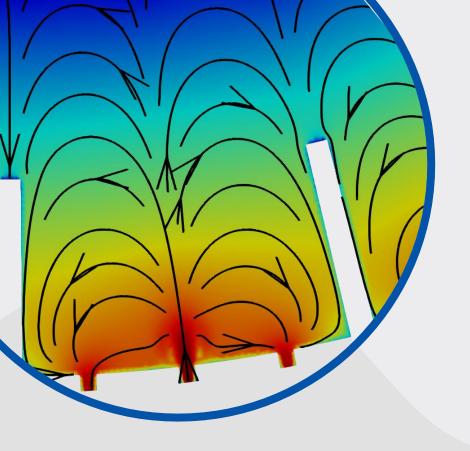
$$\mathcal{H}(u^+,u^-,n)\llbracket v\rrbracket$$

 $\nabla_h oldsymbol{v})$







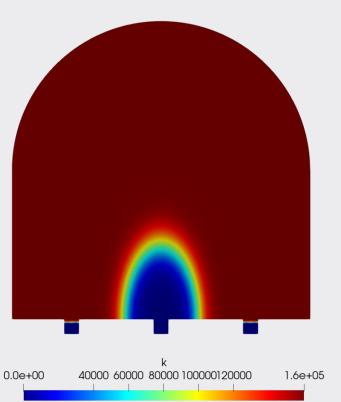


O2
Blood flow



Single placentone



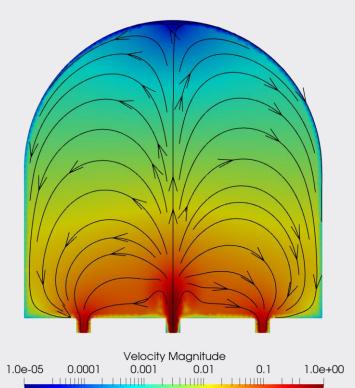


$$Re = 1 \times 10^3$$

$$\frac{1}{Dr_{max}} = 1.6 \times 10^5$$

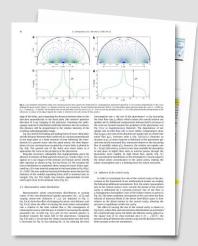


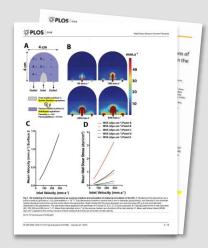
Single placentone





- Qualitatively matches other models
- Assumes no 'spilling'
- Exponential fluid slow down





[Chernyavsky et al., 2010]

[Lecarpentier et al., 2016]



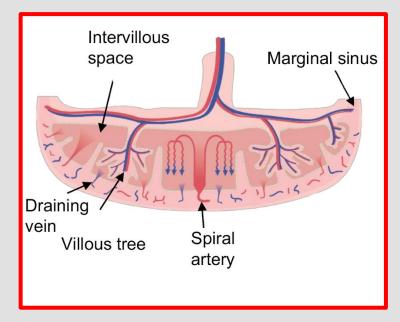
2D slice of whole placenta

100000

- 50000



- Agree with placentone simulations
- Show 'spilling'
- Include marginal sinuses



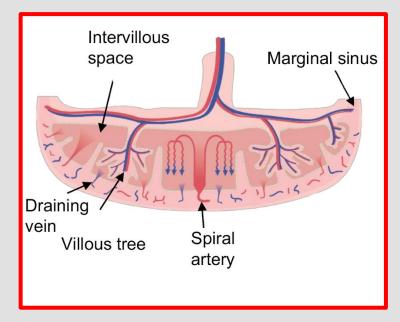
[Dellschaft et al., 2020]



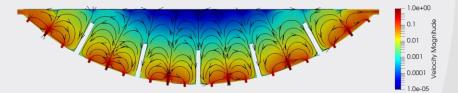
2D slice of whole placenta



- Agree with placentone simulations
- Show 'spilling'
- Include marginal sinuses



[Dellschaft et al., 2020]

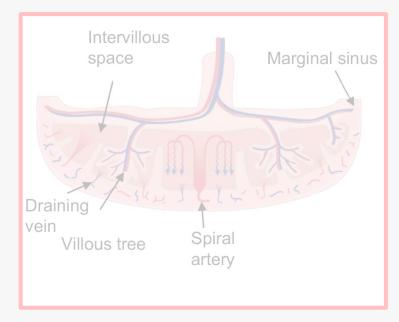




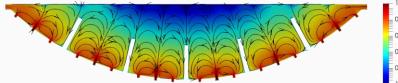
2D slice of whole placenta



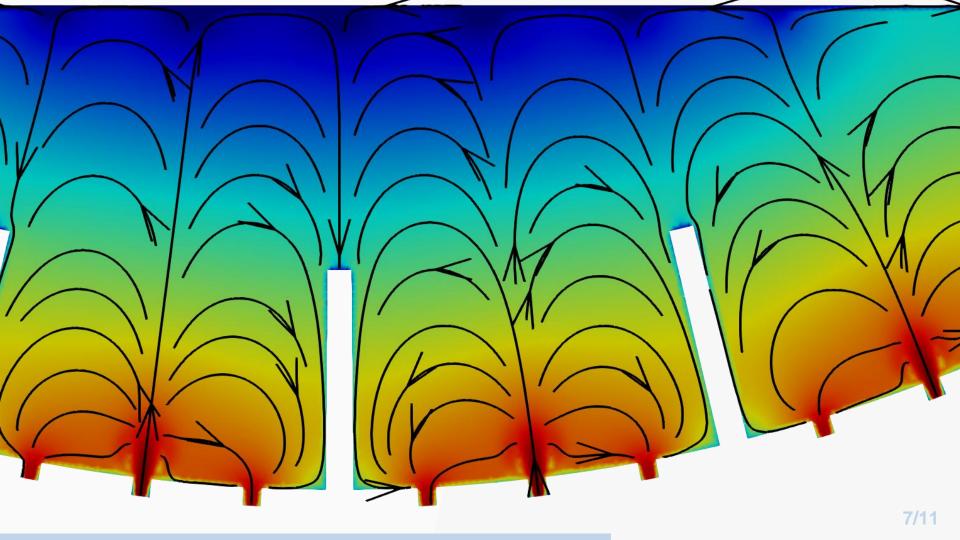
- Agree with placentone simulations
- Show 'spilling'
- Include marginal sinuses



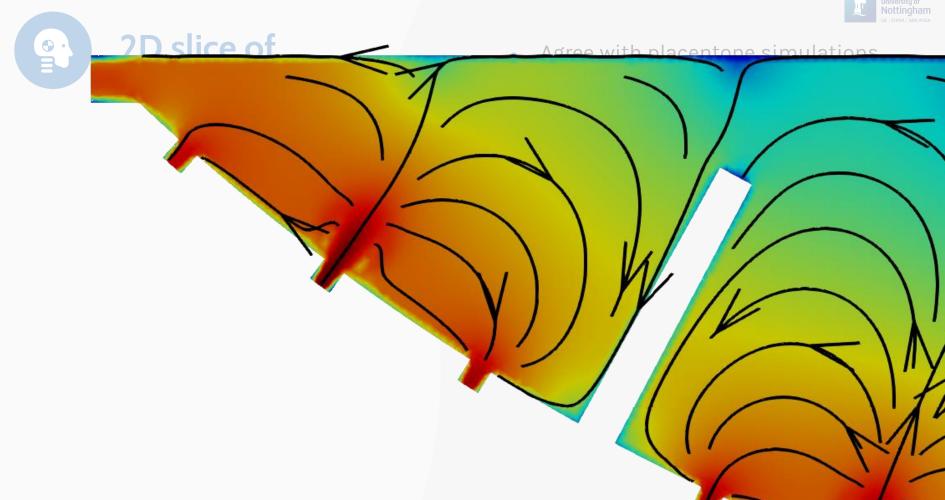








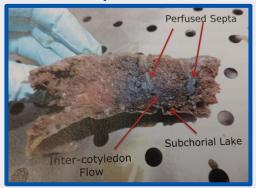




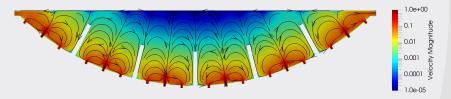


Septal veins

[Lopa Leach]



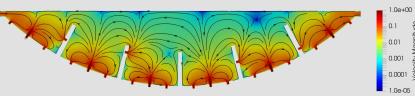
No septal veins

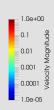


Reduces slow-moving blood

- Geometrically more accurate
- Uniformity of exchange

With septal veins





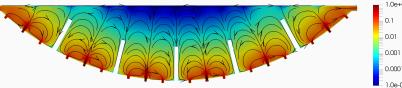


Septal veins

[Lopa Leach]



No septal veins

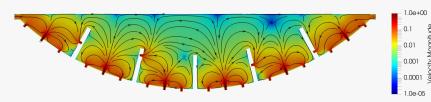


0.001 0.0001 💆



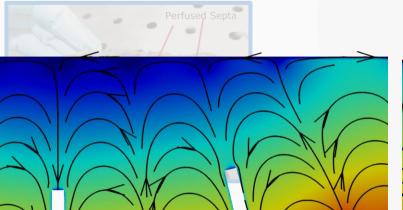
- Reduces slow-moving blood
- Geometrically more accurate
- Uniformity of exchange

With septal veins

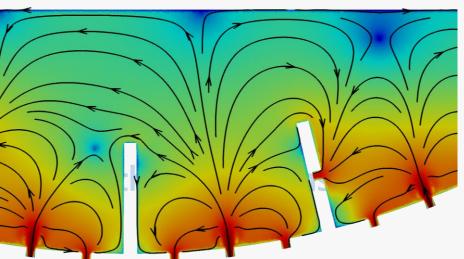








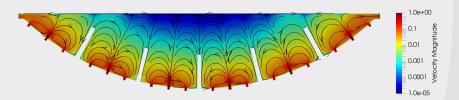
- Reduces slow-moving blood
- Geometrically more accurate
- Uniformity of exchange

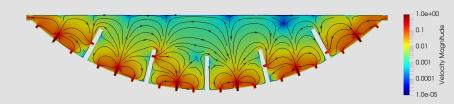






$\int \mathbf{u} \cdot \mathbf{n} \, \mathrm{d}x$

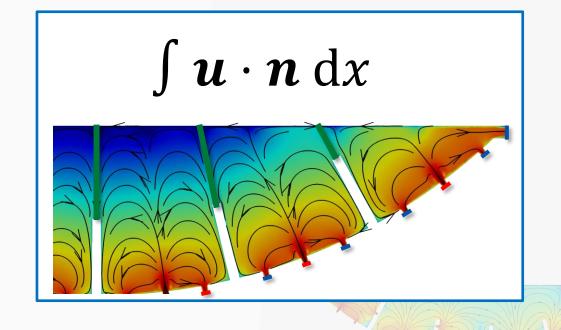






Flux transport

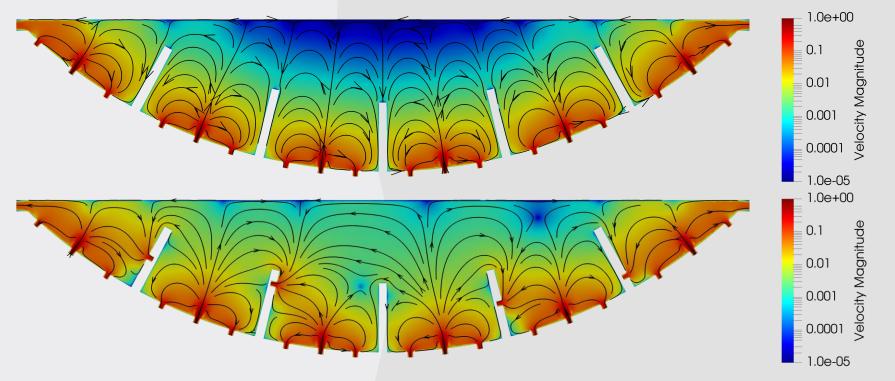








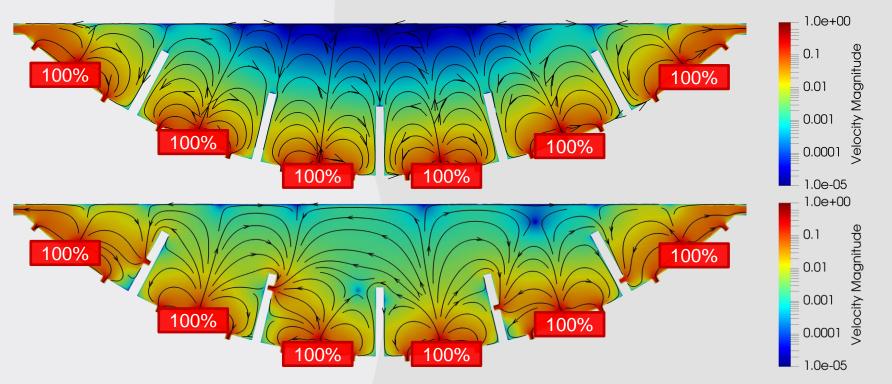
$\int \boldsymbol{u} \cdot \boldsymbol{n} \, \mathrm{d}x$







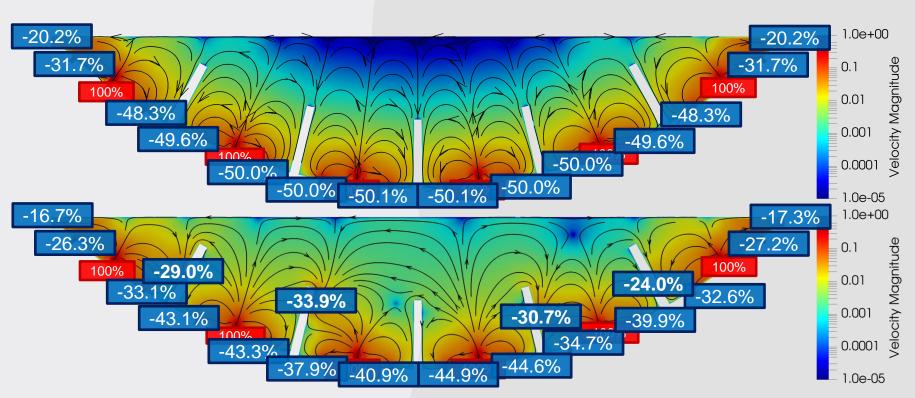
$\int \boldsymbol{u} \cdot \boldsymbol{n} \, \mathrm{d}x$







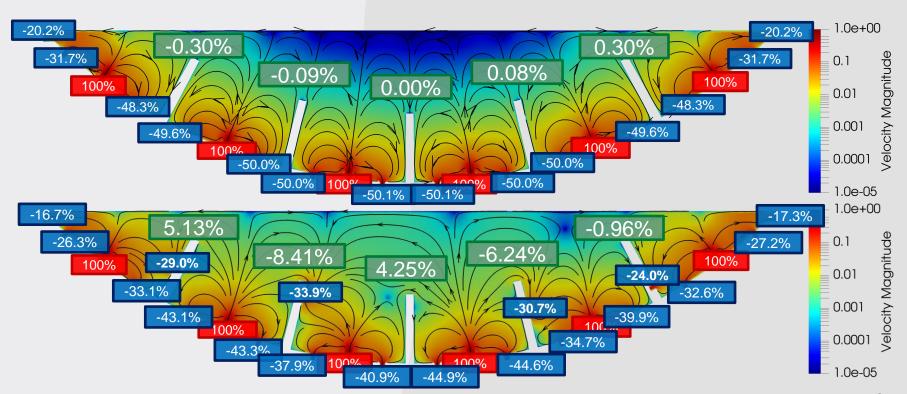
$\int \boldsymbol{u} \cdot \boldsymbol{n} \, \mathrm{d}x$



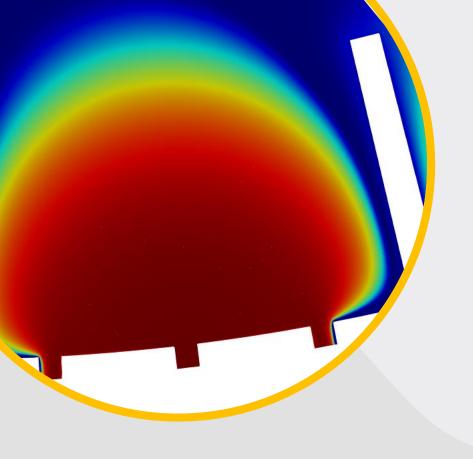




$\int \boldsymbol{u} \cdot \boldsymbol{n} \, \mathrm{d}x$







03 Nutrient transport



Nutrient transport



Reaction-advection-diffusion:

$$\frac{\partial c}{\partial t} - \frac{1}{\text{Pe}} \nabla^2 c + \nabla \cdot (\boldsymbol{u}c) + \text{Dm } c = f$$

$$c_{\text{inlet}} = 1$$

$$c_{\text{neumann}} \equiv \boldsymbol{n} \cdot (\boldsymbol{u} \nabla c) = 0$$

Oxygen parameters:

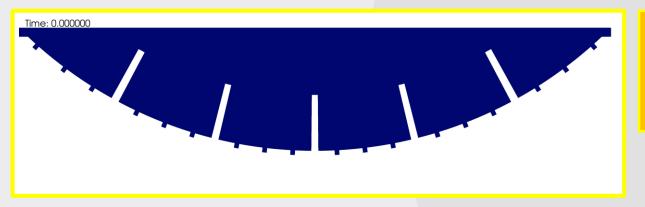
$$\frac{1}{\text{Pe}} \approx 4.17 \times 10^{-7}$$

$$\text{Dm}_{\text{max}} \approx 6.67 \times 10^{-3}$$



Nutrient transport





Reaction-advection-diffusion:

$$\frac{\partial c}{\partial t} - \frac{1}{\text{Pe}} \nabla^2 c + \nabla \cdot (\boldsymbol{u}c) + \text{Dm } c = f$$

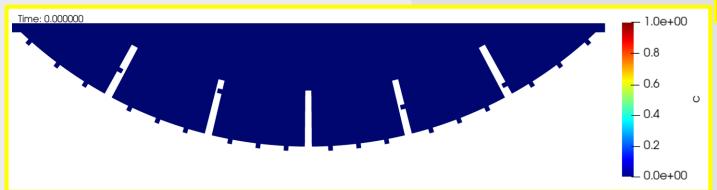
$$c_{\text{inlet}} = 1$$

$$c_{\text{neumann}} \equiv \boldsymbol{n} \cdot (\boldsymbol{u} \nabla c) = 0$$

Oxygen parameters:

$$\frac{1}{\text{Pe}} \approx 4.17 \times 10^{-7}$$

$$\text{Dm}_{\text{max}} \approx 6.67 \times 10^{-3}$$







04
In progress









VALIDATION

Compare to MRI data: particle simulations



3D

Limited 3D simulations: comparison to 2D



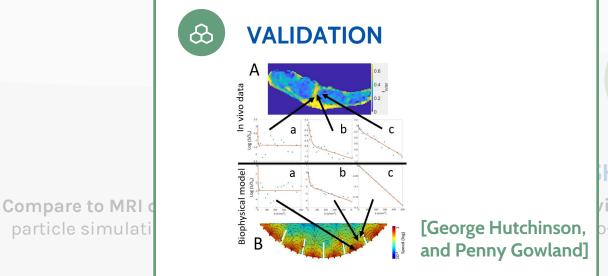
MESH MOVEMENT

Moving boundaries: utero-placental pump





Next steps



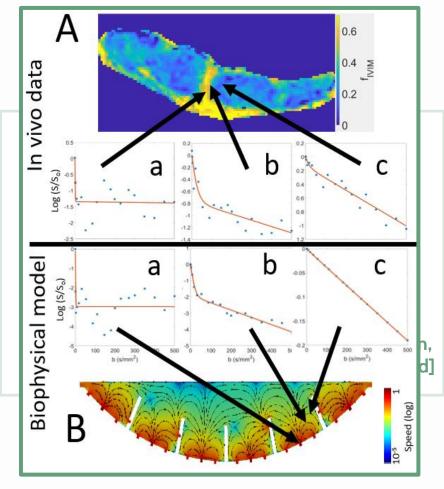


H MOVEMENT

ring boundaries: p-placental pump





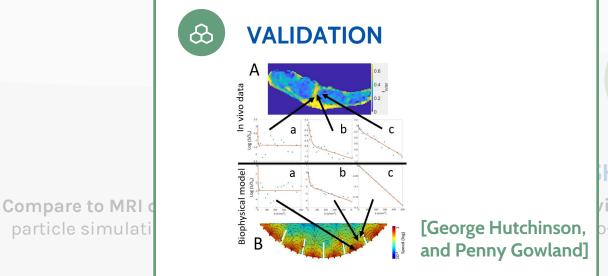


compare to MRI of particle simulati





Next steps





H MOVEMENT

ring boundaries: p-placental pump









VALIDATION

Compare to MRI data: particle simulations



3D

Limited 3D simulations: comparison to 2D



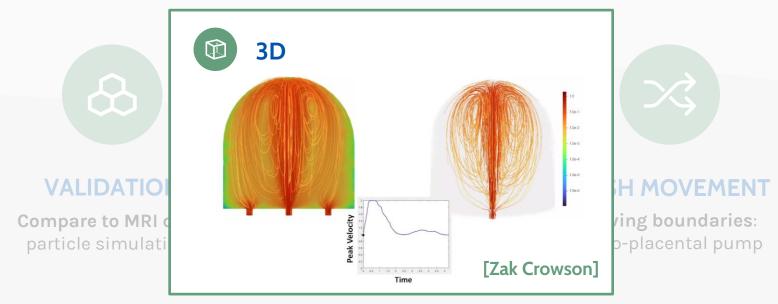
MESH MOVEMENT

Moving boundaries: utero-placental pump





Next steps











VALIDATION

Compare to MRI data: particle simulations



3D

Limited 3D simulations: comparison to 2D



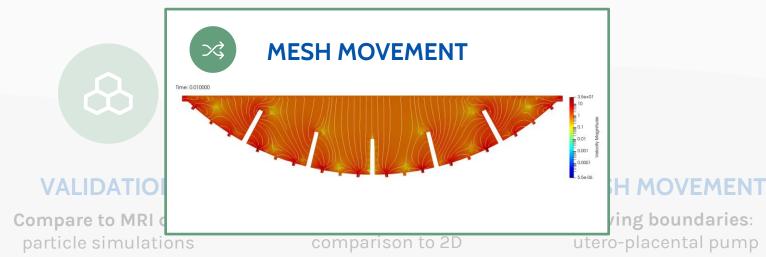
MESH MOVEMENT

Moving boundaries: utero-placental pump





Next steps

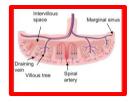




Summary

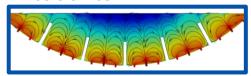


Introduction



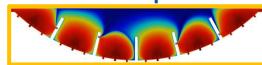


Blood flow





Nutrient transport





In progress





Questions?

Thanks for your attention!

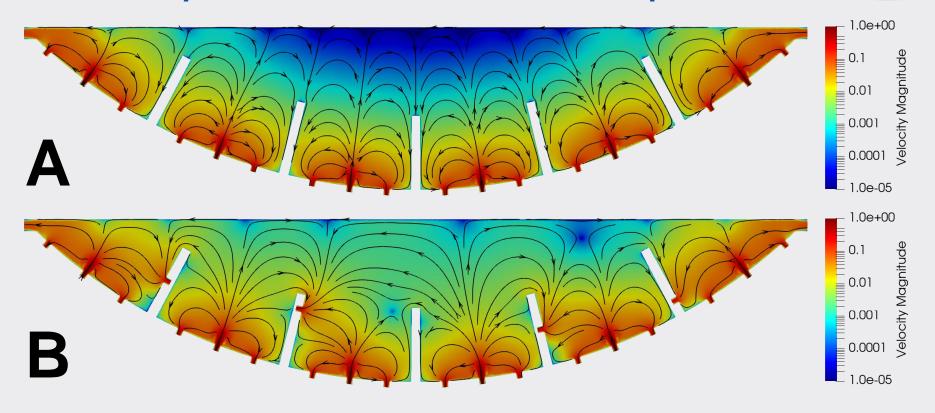
- https://adam.blakey.family
- **3** @amblakey
- ⊠ adam.blakey@nott.ac.uk







Side-by-side flow with and without septal veins



Nondimensionalised equations with parameters

Navier-Stokes + Darcy:

$$-\nabla^2 \mathbf{u} + \operatorname{Re} \nabla \cdot (\mathbf{u} \otimes \mathbf{u}) + \frac{1}{\operatorname{Dr}} \mathbf{u} + \nabla p = f$$
$$\nabla \cdot \mathbf{u} = 0$$

$$u_{\text{inlet}} = \frac{(R^2 - r^2)}{R^2} \widehat{y}$$

$$u_{\text{Wall}} = 0$$

$$u_{\text{neumann}} \equiv (\nabla u - p\underline{I}) \cdot n = 0$$

$$Re = 1 \times 10^3$$

$$\frac{1}{Dr_{max}} = 1.6 \times 10^5$$

$$\frac{1}{\text{Pe}} \approx 4.17 \times 10^{-7}$$

$$\text{Dm}_{\text{max}} \approx 6.67 \times 10^{-3}$$

$$\frac{\partial t \quad \text{Pe}}{c_{\text{inlet}} = 1}$$

$$c_{\text{neumann}} \equiv \boldsymbol{n} \cdot (\boldsymbol{u} \nabla c) = 0$$

Reaction-advection-diffusion:

$$\frac{\partial c}{\partial t} - \frac{1}{\text{Pe}} \nabla^2 c + \nabla \cdot (\boldsymbol{u}c) + \text{Dm } c = f$$

$$c_{\text{inlet}} = 1$$

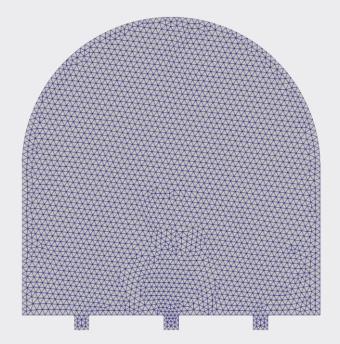
$$c_{\text{neumann}} \equiv \boldsymbol{n} \cdot (\boldsymbol{u} \nabla c) = 0$$

Re :=
$$\rho LU/\mu$$

Dr := k/L^2
Pe := UL/D
Dm := RL/U



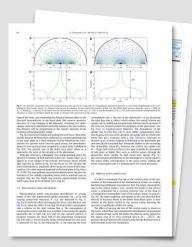
Single placentone

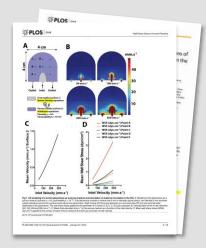


#elements = 5 736



- Qualitatively matches other models
- Assumes no 'spilling'
- Exponential fluid slow down





[Chernyavsky et al., 2010]

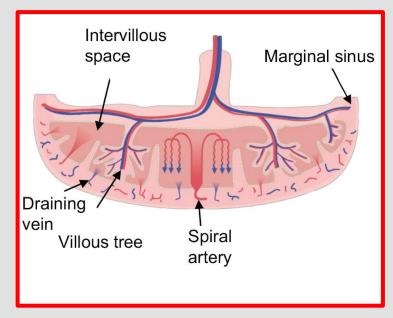
[Lecarpentier et al., 2016]



2D slice of whole placenta



- Agree with placentone simulations
- Show 'spilling'
- Include marginal sinuses



[Dellschaft et al., 2020]